Assessing the Risk of Bankruptcy

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Abstract. This paper analyzes the risk of bankruptcy in a data set of manufacturing firms from the Marche region of Italy. These firms have been defined in the past as "excellent enterprises" in terms of their size in relation to turnover and employment and are representative examples of what has become known as "fourth capitalism". These enterprises have been selected and analyzed in the course of the last fifteen years in terms of their economics, profitability, asset, financial and market dynamics in research undertaken by the *Laboratorio Fausto Vicarelli* at the Faculty of Economics of the University of Macerata.
Alternative methods of predicting the probability of default have been tested on accounting data from these particular operating units. After testing the adequacy of Altman's Z-score model and some its possible variants, new explanatory variables related to bankruptcy risk are put forward and an alternative fixed effects panel data approach is used to estimate the model parameters.

1. Introduction

The composite nature of the Great Recession which is now underway has created space for a great variety of interpretations and analyses of the issue. Developing in the United States as financial turbulence, way back on 8th August 2007, it then changed to become a financial crisis which was, first of all, full-blown and then systemic becoming a hitherto-unknown real crisis. Throughout its course, much has been written, repeatedly, about the different components which make up the credit industry, with different levels of relevance and different emphases.

The vulnerability of the various countries which were exposed to real, financial and credit transmission mechanisms in the great recession, is closely correlated to the specific situation of each of them. On one hand, the comparative fragility of the various banking and financial systems, the support lines which had been set up and the mechanisms and the extent to which they were employed have had a different impact on the real economy. On the other hand, the different levels of industrialization of countries, the specific weight of the manufacturing sector, their morphology in terms of manufacturing sector activity and the size of enterprises, their greater or lesser dependence on external sources of finance are at the basis of the greater or lesser resilience of industrial systems.

The Italian economy has had to face relatively fewer difficulties than its European partners and international competitors in terms of the stability of its financial infrastructure[1]. In contrast, it has seen a greater fall in production and gross national product than the other industrialized countries[2].

Recession, depression and crisis are macroeconomic categories. Corresponding to them at the level of the system of production of goods and services, in macro-sectors and within them in branches of economic activity, are destruction of technical and immaterial capital assets as well as that of working capital of production units which comes about when businesses fail. The empirical evidence shows that, from this point of view, the Italian economy is subject to a worrying sequence in the number of bankruptcies. The variations in percentage terms of defaulting firms rose constantly from the third quarter of 2007 to the first quarter of 2010 and it rose by double figures[3].

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[1] Ricerche & Studi (2010).
[2] Banca d'Italia (2010).
[3] Cerved Group (2010).

Evaluating the risk of insolvency is certainly a central issue for economic and financial analysis but, despite that, no mainstream for it yet exists. Institutional factors peculiar to each country and a large variety of causes which can lead to the failure of an enterprise, in fact, obstruct the route to a theory which can be applied generally. Despite this, it is an issue which must be faced, not only because it is central to credit management by banking operators, but also, after all, for its overall impact on economy.

Talking about bankruptcy risk entails talking about the Z-score. Following Altman implies contextualizing analysis to a data set of information and to its construction which must take account of institutional rules.

Below, we have highlighted the difficulties which can be expected from discriminant analysis in terms of prediction and the necessity to improve and go beyond it. The technique of estimating bankruptcy risk probability proposed, fixed effect panel analysis, allows us to solve some of the inherent problems with discriminant analysis and to obtain statistically better results (Section 2).

We referred to a closed set of enterprises from the Marche region of Italy selected in 1998. These enterprises were pinpointed as “excellent enterprises”, which can today be traced back by size and importance to those which make up fourth capitalism[4] (Section 3).

The results of the methodologies applied to the data set are presented in Section 4 together with a sensitivity analysis; Section 5 concludes.

2. A proposal for evaluating the probability of insolvency

This work takes as its starting point the first model put forward by Altman in 1968, since this is the point of reference for most credit risk models before moving on to some variations on it.

2.1. Altman’s Model

Applying a multivariate discriminant analysis to estimate the coefficients of explanatory variables, Altman tried to predict the probability of bankruptcy using five specifically selected balance sheet indicators[5] which describe the financial and economic system of a firm (liquidity, asset solidity, profitability, financial structure and productivity).

The Z-score is defined as follows:

$$Z = 0{,}012\, x_1 + 0{,}014\, x_2 + 0{,}033\, x_3 + 0{,}0006\, x_4 + 0.999\, x_5$$

where:

x_1 = Working Capital / Total Assets;
x_2 = Retained Earnings / Total Assets;
x_3 = EBIT / Total Assets;
x_4 = Market Value of Equity / Book Value of Total Debt;
x_5 = Sales / Total Assets.

[4] Colli (2002).

[5] In detail, after compiling a list of 22 potential explanatory variables classified into five standard categories, he opted for the following procedure in order to select the five final independent variables:

1. observing the statistical significance of various alternative functions (F test) including the determination of the relative contribution of each independent variable (t test);
2. evaluating the level of correlation between the explanatory variables;
3. observing the accuracy of the various profiles;
4. considering analysts’ judgements as subjective data.

If

$Z \leq 1.8$	the probability of bankruptcy is extremely high;
$1.8 < Z \leq 2.7$	it is quite likely that the firm will become bankrupt;
$2.7 < Z \leq 3$	some caution is needed, but it is unlikely that the firm will become bankrupt;
$Z > 3$	the firm is unlikely to default.

The Z-score coefficients, along with the ranges quoted above, were calculated on the basis of a sample of 66 manufacturing corporations. The firms were divided into two groups of 33. The first group was made up of firms which had filed for bankruptcy in the years between 1946 and 1965, spread relatively uniformly over the twenty-year period; the data referred to the financial period immediately prior to bankruptcy. The second group was made up of non-bankrupt manufacturing firms which were randomly selected from the Moody's list and other sources and stratified both by sector and size[6]. The data relating to the firms in this second list were taken from the same years as those in the first group.

Using an F-test, Altman concluded that, for classification purposes, the most relevant variables are x_3 (profitability index), x_5 (productivity index) and x_4 (financial structure index), while x_1 (liquidity index) is the least relevant.

The model proved extremely accurate when it was tested on a sample of firms different from those used to estimate the parameters for the year immediately prior to failure; type I error (failure of a firm when bankruptcy was not predicted), the most dangerous, and type II error (firms which had been expected to go bankrupt which did, in fact, not) were both modest.

Naturally, the effectiveness of the model in classifying enterprises is reduces gradually as we go back in time.

Altman later suggested using the same model for privately held firms by modifying the x_4 variable that becomes Net Worth / Total Debt[7].

2.2. Potential and limitations of Altman's model and alternative solutions

In Altman's initial work, and in the later analyses it inspired, equal subsample group sizes (50-50) were used; this choice was dictated by a number of factors including the need to reduce variance in coefficient estimation, the need to identify the differences between groups more efficiently and the desire to contain the costs of selecting the firms whose data is to be used in the model.

This balance between non-failing and defaulting firms is evidently in opposition to the concept of random sampling, since it can only be based on *ex post* knowledge of some characteristics of the firm meaning that the predictive capacity of the model is conditioned in an *ex ante* decision-making context.

In theory, the numerosity of the subsets should, in fact, reflect the non-failing/defaulting composition of the population; this is the only way in which the estimated model can be applied directly to the real context with probability which coincides *a priori* with the sample without having to make further adjustments.

One peculiarity of Altman's model is that it considers the parameters estimated by the discriminant function valid, and therefore constant, for long periods of time. This aspect is, in reality, closely connected to the *ex ante* and *ex post* predictive capacity of a model and, although maintenance of the diagnostic capability over time is doubtless based on the stability of the relationships observed between the explanatory variables and bankruptcy, it is also the case that it is

[6] Small and very large enterprises were excluded from the sample of healthy firms for the purposes of homogeneity with respect to the size of the corporations in the first group.
[7] Altman (1993).

necessary to verify a model's performance periodically and perhaps go on to re-estimate the parameters when the discriminant analysis effectiveness tends to be reduced as national banks suggest.

The selection of explanatory variables in Altman's model is the result of a purely empirical research process with adaptations which are often dependent on individual choices and consistent with the basic absence of a solid firm bankruptcy or crisis theory. The problem has therefore been raised of testing whether it is still worth using the same variables in a different space-time context or, rather, whether they are still really able to classify this type of enterprise efficiently, minimizing the type I and type II errors.

One of the most hotly-contested issues in the scientific debate since the publication of Altman's research has been what choice of statistical method to make when designing models for bankruptcy risk prediction. Let us, therefore, consider a brief overview of the potentiality and limits of using the most widespread methodologies in order to arrive at a new proposal relative to the case in point.

Discriminant analysis aims at seeking out the optimum combination of indicators which can best separate two sets; in particular, in our context of analysis, such a methodology implicitly assumes that the observable firms come from two given distinct universes and that detecting variables in the balance sheet can help find features which are relevant to finding the actual universe to which each enterprise belongs. In looking for a single signal which would allow us to consign the statistical unit to one of two groups *a priori*, each indicator, considered more as a signal of status than a quantitative proxy of the real economic and financial situation of the firm, is necessary in virtue of the contribution it makes to the overall signal. This methodology is, in essence, based on the assumptions that the indicator distribution is of a multivariate normal type, that the averages of the balance sheet indices of the two groups are significatively different and on the equality of the variance-covariance matrix of the two groups.

As far as the first assumption is concerned, it is a well-known fact that indicator distribution is not of normal type and that, in any case, it is not certain that their conjoint would be the same. Given that this circumstance has an inevitable influence on the test of significativity and on the performance level of the models, some researchers have proceeded to transform the original variables[8], if only to make the distributions more symmetrical. In doing this, they have run into further problems involving change in the relationships between the explanatory variables and consequent difficulties in interpreting the functions obtained as well as in alterations in the coefficient estimates. Actually, the problem of non-normality in the indicator distribution could be overcome since numerous experiments have shown that the effects of violating normality in discriminant analysis (both linear and quadratic) in this particular context would generally be less dramatic than statistical theory tends to suggest (Altman *et al*., 1981).

If dispersion between the groups differs, meaning the fall of the last of the above-quoted assumptions on which the discriminant analysis is based, the tests on the differences between the medians of the groups[9] would inevitably be influenced and it would seem preferable to turn to a quadratic discriminant function even though this, as empirical evidence teaches us, would not be more systematically relevant[10] or efficient than a linear discriminant function. The latter has proved globally better than the former (Altman *et al.*, 1977). The empirical evidence also shows that a quadratic discriminant function, that must inevitably take account of the interactions among the variables and of the quadratic terms, is in any case characterized by a high number of coefficients

[8] Where possible (only for positive values of the indicators) the logarithmic operator or the square root has often been used; sometimes, however, the reciprocal of the values or the transformation of the original values into classes is used.

[9] This problem is specific to the approach with discriminant analysis, which is based on the assumption that there are two distinct populations and it does not apply to linear or logistic multiple regression, which are based on the assumption of a single population.

[10] It would actually seem (Eisenbeis, 1997) that the elements which could influence the final outcome would be due to the effective entity of the difference between the variance-covariance matrices, to the number of elements which make up the samples and to the number and type of the balance sheet indicators used.

which are, amongst other things, often difficult to interpret. In other words, although the starting point is a model designed on the basis of a limited number of explanatory variables, within a quadratic structure many coefficients would be generated and so it would be difficult to understand from an economics and financial point of view if they would be associated with a sign which would not be coherent with theoretical expectations. Although the economic plausibility of the coefficient signs and, as a result, the right direction for the cause and effect relationship is undoubtedly a basic condition for acceptance of any model, the modest improvements obtained by using quadratic functions which tend to go hand in hand with probable sign errors in a significative number of coefficients can only lead to the model being rejected from an economics point of view. In addition, the coefficients of the same variables estimated using a linear function would, in contrast, maintain the correct signs and would thus explain why it is the preferred type of function in many applications (Varetto, 1999).

A further difficulty, by no means easy to solve, which is encountered when implementing a bankruptcy prediction model based on discriminant analysis concerns evaluation of the relative importance of the function variables, or rather their individual discriminatory capacity; in this context, in fact, the coefficients are non-unique, as only the ratios between them is so, making it applying tests to check out alternative assumptions pointless.

To conclude this brief overview of discriminant analysis and the different ways it can be used when defining a bankruptcy prediction model, it would be interesting to note that other empirical studies (Eisenbeis, 1997) have highlighted how linear analysis has shown itself robust to the violation of the methodological assumptions only when large-scale samples are used.

Multiple regression has also been used in some research on credit risk: in our research context, the approach mainly consists in estimating a model in which the dependent variable is a qualitative dichotomic (which describes belonging to the set of non-defaulting or defaulting firms) or a probability of default, while the balance sheet indicators are the explanatory variables[11]. It therefore seems clear how, with this type of model, the degree of the firm's economic and financial difficulty has to be estimated: in a multiple linear regression aimed at defining a model for the evaluation of credit risk the assumption is in fact that firms are randomly extracted from a single universe and an attempt is made to estimate their level of health (or the linear probability of insolvency/bankruptcy); the state of health can therefore be considered a characteristic expressed through a continuous latent variable which has only two observable essential extreme determinations (zero and one). It follows that this type of model makes the basic assumption that there is a causal relationship between the variables observed on accounting data and the dependent variable, implicitly supposing that there is a cause and effect relationship between the economic phenomena distilled from the balance sheet variables and the firm's state of health. It can be shown (Maddala, 1983 and 1992) that multiple regression has a close relationship with discriminant analysis: linear discriminant function coefficients are in fact equal to those from an OLS regression, linear in type, unless a constant relationship.

Use of multiple regression is not without its critics. Some difficulties exist, although they are not, in reality, insurmountable. A first problem is, in actual fact, connected to heteroscedasticity, while the second difficulty regards the fact that estimating the independent variable often does not determine values equal to or included between zero and one, as it would be logical to obtain in order to interpret the results in terms of non-defaulting or defaulting enterprises or, in the case of linear probability models, in terms of the probability of being, more or less, in a state of distress.

The former problem is easily removed by using, alternatively, a two-stage estimation procedure, despite the fact that this would be unable to solve other issues such as, for example, the absence of error normality, or rather, as we shall suggest, by means of linear panel regression. Obtaining monotonically dependent variable values within the zero-one interval could also be

[11] In this context, the balance sheet indicators are therefore the exogenous variables which are able to explain the real economic and financial situation of the enterprise, its evolution and any deterioration leading to a state of crisis.

achieved by using a logistics model (*logit*)[12] or, much more easily, as we shall see in our implementation, by normalizing the balance indicators.

As concerns the use of a logit model to obtain dependent variable values which are monotonically within the zero-one interval, it must be borne in mind that the assumption about the shape of the distribution of bankruptcy probability is actually very strong and could heavily influence the results obtained from its implementation. A further relevant problem which would be encountered by using a logistic model concerns the perception of changes in probability given that the morphology of the function itself ensures that there are very different variations along the various sections of the curve[13] in contrast with what happens in a linear relationship where the slope is known to be constant. A further factor which could lead to a decision not to opt for a logistic multiple regression lies in the fact that logistic regression generally produces results which are not very different from those obtained by the more tried-and-tested linear discriminant analysis[14].

In the light of the above, in order to solve the problem of dependent variable values of a multiple linear regression outside the zero-one interval, it would seem much more immediate and effective to turn to normalization of the balance sheet indicator values with respect to the opportune maximum values, without the need for strong basic assumptions and without creating any problems in interpretation of the changes in probability.

In relation to the evaluation of the relative importance of the function explanatory variables in linear or logistic multiple regressions, in contrast with what happens, as we have seen, when discriminant analysis is applied, the problem could be easily resolved by the well-known parametrical statistical tests[15].

The normality of the indicators' distributions is an assumption which is relevant not only, as stated above, to discriminant analysis, but also to linear parametric techniques; this circumstance, we would like to restate, generally influences both tests of significativity and the performance of the models. However, just as in the case of discriminant analysis, empirical evidence has shown that the effects of the violation of normality on linear and logistical multiple regression are not generally as consistent as statistical theory tends to underline (Altman *et al*., 1981).

Over time, the literature has been broadened with numerous studies aimed at looking deeper into the various methodological aspects and different approaches as well as experimenting with new techniques which have often come from the field of artificial intelligence[16]. Discriminant analysis and multiple regressions continue, however, to be of great interest to those operating in banking

[12] The key idea of a logit model is that it supposes that there is a relationship between the probability of the firm becoming bankrupt (non-observable variable) and a set of observable signals which are closely connected to bankruptcy; that which can actually be observed is only a dichotomic realization of the probability of bankruptcy. In reality, the logit model is not the only one which is able to produce values in the zero-one interval, but some mathematical properties make it preferable as is more easily tractable and, therefore, more suitable for application. A very similar model to logit which is implemented in credit risk analysis is probit model; this is based on the key assumption regarding the shape of the cumulative distribution of bankruptcy probability which, instead of being cumulative logistic is assumed to be cumulative standardized normal. Despite the cumulative distribution shapes being different, the results provided by the logit and probit approaches in applications inherent to defining models for credit risk analysis are extremely similar; undoubtedly, however, normal distribution presents greater difficulties in terms of mathematical tractability than logistical which is therefore preferable for the purposes of implementations.

[13] In detail, movements along parts of the function near to the horizontal asymptotes generate perceptions of changes which are very limited in probability even when alterations in the firm's situation are major; in the central area (oblique) of the logistic function, however, relatively small changes in the firm's situation translate into relevant variations in probability.

[14] See Bardos and Zhu (1998).

[15] The literature on the subject includes various methods which have been suggested for assessing the importance of explanatory variables. In particular, we remind the reader to the approach which, bearing in mind existing relationships which have been described between linear discriminant analysis and regression of the same type, after setting up a model with the former methodology proceeds to a new estimation of the explanatory variable coefficients with the second, with the one aim of assessing the individual efficiency of the variables.

[16] See Altman, *et al.* (1994), Carrara and Cavalli (1996), Baetge and Uthoff (1998), Piramuthu (1998) and Quaranta (2008 and 2009).

who, in the light of new opportunities offered by the rules, are continually in search of new methods to supplement traditional procedures in credit selection[17].

There have recently been a number of contributions where the classification into the subsets non-defaulting and defaulting firms is obtained by employing a number of approaches at the same time (mainly linear discriminant analysis and logistic multiple regression) and then choosing the model with the best performance out of the sample, testing, by this way, non sample-specific results derived by an overfitting and the consequent inability of the model to provide generalizations.

In the light of what has been said concerning the choice of statistical method to adopt when setting up predictive models and bearing in mind the availability of space-time data, in our empirical analysis we have decided to use two methods together: firstly, linear discriminant analysis, a paragon which cannot be substituted since it is still the most implemented worldwide and because of the legacy which has stemmed from the first model put forward by Altman; the second method is a panel fixed effects linear regression. In using this new methodology, an attempt has been made (i) progressively to enucleate the factors which can influence the critical aspects of business management most likely to be more correlated to probability of default and (ii) to analyze and test the coherence of the empirical evidence with theoretical expectations in relation to causal direction (parameter's sign) of every explanatory variable held to be econometrically significative, at the same time as solving the widely-debated problem of heteroscedasticity in multiple linear regression and, as we shall see in detail below (Section 4.4), avoiding obtaining values of the dependent variable which are outside the zero-one interval by means of an opportune normalization of the values assumed by the balance sheet indicators in space and time.

Panel data, as is widely known, are characterized by a greater wealth of information both with respect to historical series and to cross-sectional data and, wherever they are available, they are to be preferred also because the information which is supplied from the temporal dimension is able to contain to a great extent, if not eliminate completely, the problems of heteroscedasticity which occur in multiple regression.

In contrast with the analyses carried out exclusively in a spatial dimension, temporal data (within or intra-individuals) are in fact considered. They permit an answer to the question (considering the characteristics of the firm as given) of whether institutional events or policy changes can have a certain effect on the relationship being analyzed over time. On another front, in contrast with analyses which are purely based on time series, data panels also consider information grasped by each individual (between or inter-individuals) which, in our context of analysis, (considering as given institutional events or changes in policy over time) allow us to answer the question about whether the specific characteristics of the considered firms can have an effect on the relationship being analyzed.

With a panel data, therefore, more complex assumptions can be investigated (concerning dynamics, but also micro and macroeconomic characteristics). The high number of observations (from N or T to $N*T$) also permits both a better estimate of parameters and a more appropriate use of asymptotic statistical properties[18].

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[17] The same Regulatory Bodies have directed increasing attention to methodologies in order to obtain a more efficient and effective credit selection; to this end, it would seem interesting to note the contributions of Laviola and Trapanese (1997) and di Marullo Reedtz *et al.* (1996). The European Committee of Central Balance-Sheet Data Offices has compared the different experiences which have come from using discriminant analysis for the purposes of bankruptcy risk diagnosis in the main European countries.

[18] In further detail:

(i) reduced problems of collinearity, as a result of wider individual variability, make it possible to obtain more efficient estimates and a considerable improvement in the capacity to discriminate between different assumptions (tests);

(ii) the introduction of non-observed effects makes it possible to obtain specific individual/temporal heterogeneity and reduces the problems connected to aggregation; distortion due to omitting individual or temporal explanatory variables (correlated with the explanatory variables included) can be reduced;

This wealth of information to use more than one estimation strategy, in such a way that the parameters in object can be identified using the variability of data in a time dimension, cross-sectionally or both.

3. Reference data

In Italy, the only one region having more than 40% of persons employed in industry as a proportion of those employed in the non-financial business economy is Marche; this evidence is more important if we consider that in Europe only 17% of regions reaches more than share (Eurostat, 2009).

The economy in the Marche region of Italy is characterized by widespread entrepreneurialism, quantifiable in terms of one enterprise for every 10 inhabitants of the region.

15% of firms operate in the manufacturing sector, providing employment for 27% of the working population, while contributing 26% of the regional added value. Firms mainly operate in the so-called traditional sectors and are mainly small (87% have fewer than 10 employees). Large enterprises[19] represent only 0.1%, whilst medium-sized are 1.3% of the total. In employment terms, the medium-large enterprises represent 29% and this alone could be enough to give those companies an importance to the area in which they operate which is greater than it seems in purely numerical terms. When local businesses with more than 20 employees are added, the employment share they cover reaches 46%, almost half the entire manufacturing segment.

Their relevance in terms of employment is not the only reason why we have chosen to select some of these firms. There are some firms among them which are benchmarks in the sector certainly for size, in terms of both number of employees and turnover, but also for innovative strategies and quality in production. Therefore, these firms can be also considered representative examples of what in Italy has become known as “fourth capitalism”[20].

We started from a closed set of 200 manufacturing firms, which had already been selected in the year 1998 in order to analyze, over time, the performance of medium-sized enterprises from the Marche and their impact on the local economic system. This is a closed group made up of capital firms, derived from a series of stratifications which consider, on one hand, the size of the enterprise on the basis of turnover and, on the other, specialization in the sector and presence in the territory.

The set was updated gradually until 2008. Data updating has necessarily meant change in the original numerosity of the set: its closed nature implies that any enterprise can leave if it is taken over by an outside enterprise causing it to lose its corporate identity; in that case, to ensure homogeneity in the data, the enterprise has been excluded from the set from the start. The same choice has been made when, because of company policy, manufacturing has ceased and the enterprise has become a holding company or has been transformed into a commercial unit. Besides this, since the elementary unit of measurement is the balance sheet of each enterprise[21], there could be a problem of homogeneity and constancy in the evaluation criteria used in drawing up the accounting prospects: in case where the notes to the accounts show that, because of reorganization of production, assets or financial processes, the balance sheet of a firm is no longer comparable with those from previous years, the enterprise must be removed from the reference set. All this has meant further contraction of the sampling field leaving as original companies, since 1994, only those

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(iii) finally, individual heterogeneity can explain serial correlation and general serial dependence in the composite error term of and it can also discriminate between time-invariant and state dependent heterogeneity.

[19] The number of employees is used as dimensional variable. We take medium-sized enterprises to mean those with between 50 and 499 employees; large enterprises are those with 500 or more employees.

[20] Turani (1996), Colli (2002). “Fourth capitalism” groups together medium and large enterprises with a turnover of less than 3 billion Euros; anyway, these are conventional limits with no absolute value.

[21] Profit & Loss account and Balance sheet data have been integrated with non-accounting data highlighted in notes to the financial statements, management reports and other official public sources.

where there is perfect comparability of empirical evidence over the whole time period of the analysis.

The unusual nature of this set of companies makes it particularly suitable for our ends since it provides economic and financial information about a consistent number of medium-sized enterprises over the course of fifteen years. As the aim of the present work differs from that for which the set was created, further modifications have been made to it. In the first place, firms have been excluded if they have been incorporated internally into others as they have not ceased to exist because of a crisis, but continue to trade as part of the absorbing corporation. Furthermore, enterprises quoted on the stock exchange have been excluded, with the aim to obtain a greater homogeneity of information.

Following these further adjustments, the set of firms (henceforth the population of the Marche's "excellent enterprises") consists of 156 firms in the start year (1994) which has progressively reduced to become 125 in 2008.

4. Stages in analysis and empirical results

4.1. Implementing the Altman model: "50-50 sample"

Analysis begins with the first model suggested by Altman in 1968, reference point for most credit risk models.

First of all, the group of firms was subdivided into non-defaulting and defaulting companies.

Enterprises which have failed judicially[22] most certainly come into the second group. It is, however, necessary to remember that it is normal practice for firms to make considerable effort to escape bankruptcy between the beginning of irreversible economic and financial crisis and true bankruptcy or receivership. This can take some years, and the firm may cease trading or reduce its production drastically in order to liquidate some of its goods such as property. The firm might also sell or lease all or part of its production processes (plant, machinery, contracts and workforce). Using data from the balance sheet relating to the year before declaration of bankruptcy, highly anomalous data for an enterprise, would therefore lead to a model which discriminates firms at risk as being only those which are already in a state of crisis. This result would be of absolutely no use to financial intermediaries who are well able to recognize this level of crisis simply by reading the balance sheet. On the contrary, it is necessary to recognize the signals of economic and financial distress in enterprises which are operating normally. For this reason, the decision was taken to consider, not the last balance sheet before failure, but that relating to the final year of these firms' normal operations.

The decision was also made to widen the definition of defaulting enterprises to include those which were in voluntary liquidation.

As far as the non-defaulting enterprises are concerned, some authors have raised the issue of possible exclusion from this group of "vulnerable" firms, i.e. enterprises in good health but with elements of marked financial or economic weakness[23] in order to allow better application of discriminant analysis. This exclusion would, however, lead to an inappropriate model which would be of little use as it would raise doubts about the rather common situation of firms which find themselves in temporary difficulty. In addition, procedural problems would present themselves because of a pre-selection that uses at the beginning of the procedure some appropriate variables chosen and therefore which could not be used later in the actual implemented discriminant analysis.

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[22] In the sense of setting an official procedure in motion: declaration of bankruptcy, receivership, temporary receivership or preventive administration.

[23] Peel and Peel (1987), Gilbert et al. (1990), Johnsen and Melicher (1994).

In case in object, the decision was made to consider all companies which did not enter into the above definition of defaulting as being non-defaulting.

Once the two groups were defined, following the procedure described by Altman, a sample made up of all the defaulting enterprises (31 firms) and the same number of non-defaulting enterprises was set up (a “50-50 sample”).

For defaulting firms, as already specified, the balance sheet data relative to the final year of normal operation were considered, while in order to select the non-defaulting firms, for each failed enterprise was randomly chosen a healthy firm in the same sector, in the same year, and of similar size, considering as a proxy of the latter firstly the total assets and then the turnover.

The information in this data set, as there were no situations of criticity before 1999, refers to the years 1999-2007; on this basis, we moved on to constructing the five explanatory indicators of the probability of default suggested by Altman in relation to each firm in the “50-50 sample”.

As it was necessary to implement a multivariate discriminant analysis, considering the numerosity of the two subsamples, defaulting and non-defaulting enterprises, after conducting some preliminary sensitivity analyses into the significativity of the model solution, the decision was made to calibrate the width of the training set to 70%, for a total of 42 enterprises, and the test set to 30%, for a total of 20 units, in both cases equally divided into healthy and failed as the context was that of a “50-50 sample”.

The prediction error obtained by this procedure was considerable being 40% overall (40% type I error and 40% type II error), therefore not modest as on the contrary emerged in Altman’s implementations and moreover worrying especially in terms of the high incidence of failing to pinpoint defaulting firms.

Table 1. Test set classification results: “50-50 sample”

	Predicted Group		
Actual Group	non-defaulting	defaulting	Total
non-defaulting	6	4	10
defaulting	4	6	10
non-defaulting	60.0	40.0	100.0
defaulting	40.0	60.0	100.0
Overall classification error: 40%			

Information from 2008 on the actual state of health of the enterprises belonging to this sample, therefore confirm that this procedure is of little use in pinpointing firms in difficulty *ex ante*.

4.2. Implementing Altman’s model: “80-20 sample”

We considered whether the choice of a “50-50 sample” might be behind the previous disappointing results, as many authors have already reported[24].

Really, in fact, in a normal economic context, it is difficult to find the same number of companies which are non-defaulting and defaulting, since there are more of the former than the latter; this topic, however, appears also within our set, since an 80-20 distribution between non-defaulting and defaulting firms results.

In addition, prediction parameters calibrated to a “50-50 sample” should, as did not happen in the above application, in any case lead to a lower probability of type I error and, if anything, to a

[24] Among others, Varetto (1999).

greater II type error guaranteeing, in this way, greater caution when evaluating credit risk. This topic does not emerge from the previous results too.

Therefore, with the aim to not exclude any of the failed firms given the basic importance of their quantitative information, as we decided in relation to "50-50 sample", considering the actual distribution between non-defaulting and defaulting firms in our data set, we chose to use it entirely; from the original panel data for each statistical unit we utilized the balance sheet of a single year, reaching in this way an "information sample". Specifically, we considered all the 31 defaulting firms (now corresponding to 20%) for which we chose once again the quantitative information referring to the last year of normal operations, while in relation to the others 125 non-defaulting units, such as the residual 80%, we utilized a single year balance sheet data selected on the defaulting firms temporal distribution.

As in the previous section, the decision was taken to calibrate the width of the training set to 70%, for a total of 109 firms of which 21 failed, and the test set to 30%, for a total of 47 units of which 10 failed.

The overall prediction error obtained from this procedure was less than that of the "50-50 sample" implementation, and equal to 21.3% (40% type I error and 16.2% type II error).

Despite this improvement in the overall performance, there was no change, however, in the incidence of type I error; to reduce it, it is probably necessary to consider choosing new explanatory variables of the default probability.

Table 2. Test set classification results: "80-20 sample"

	Predicted Group		
Actual Group	non-defaulting	defaulting	Total
non-defaulting	31	6	37
defaulting	4	6	10
non-defaulting	83.8	16.2	100.0
defaulting	40.0	60.0	100.0
Overall classification error: 21.3%			

4.3. Building and selecting new explicative variables and testing the "80-20 sample"

As the previous results had been disappointing, the decision was made to proceed by testing the possibility of specifying new explicative variables for the bankruptcy of a firm, following a different path from Altman. As there was no alternative theoretical reference model in the literature which had been unanimously agreed and then implemented, on the basis of the information at our disposal 70 balance sheet indicators were drawn up and subdivided into indices of financial structure and asset solidity, liquidity, profitability, financial management and extraordinary management.

Of these indicators, 38 were selected: first of all in an attempt to favor those which could be normalized in order not to alter the effective weight of single pieces of information later, when the data were processed; in the second place, indices were excluded if they appeared to be clearly multicollinear.

As the indices are built on accounting data, they can take on values which are less than zero if the income quantities used in their structure are negative; for this reason, after normalization, indices can take on values between -1 and 1.

Building and then selecting explicative variables of the default probability we refer to the "80-20 sample" because of its best theoretical and empirical performance, as described in previous section.

Once normalization had been carried out, in order to avoid problems related to multicollinearity in later calculations, a correlation analysis was carried out[25]; from the 38 starting indicators we then arrived at 24.

Performing multiple linear regressions an attempt was made to enucleate the factors which were able to influence the critical aspects of business management correlated to the probability of default, both analyzing and testing the coherence of the empirical evidence with the theoretical expectations relative to the causal direction (parameter is sign) of each explanatory variable which was held to be econometrically significative[26].

Only three variables resulted significative and associated to a sign which was coherent with the theory:

- ROI = Operating profit / Total Assets
- Lv = Net worth / (Net worth + Funded Debt)
- CPN = Registered Capital / Net worth.

For the purposes of discriminant analysis, the width of the training set was calibrated once again to 70% and that of the test set to 30%.

The overall prediction error obtained was 12.8% (40% type I error and 5.4% type II error).

Table 3. Test set classification results:
"80-20 sample" with new explanatory variables

	Predicted Group		
Actual Group	non-defaulting	defaulting	Total
non- defaulting	35	2	37
defaulting	4	6	10
non- defaulting	94.6	5.4	100.0
defaulting	40.0	60.0	100.0
Overall classification error: 12.8%			

For further confirmation of the results obtained, we proceeded to construct another two different allocations of firms in the training and test sets and obtained results which were not significantly different.

Despite the improvement in the overall performance, once again, the high incidence of type I error is unchanged. At this point, it became necessary to use a procedure of a different nature.

4.4. The new model and the panel estimation

Starting from the same 38 indicators previously selected, in order to avoid multicollinearity problems in later calculations, a correlation analysis was carried out using the information from a panel data, in the temporal window between 1994 and 2007. This was obviously a non-balanced

[25] To this end, statistical software PASW Statistics 18, generally known as SPSS, has been used.

[26] The econometric tests carried out, including those for assessing any possibility of an incorrect model specification due to the omission of relevant variables, including the particular case of an incorrect use of the functional form, mean that the results obtained can be judged reliable.

panel data due to the progressive exit of defaulting firms, and to fill the information gaps which gradually began to appear in relation to these firms, the values relating to the last available year before failure were repeated, as is usual practice[27].

As a result of this procedure, 23 variables were pinpointed which were substantially the same as those of the final selection described in the previous section.

By means of fixed effects panel linear regressions, an attempt was made both to enucleate progressively the factors which can influence the critical aspects of business management most likely to be more correlated to probability of default and to analyze and test the coherence of the empirical evidence with theoretical expectations in relation to causal direction (parameter’s sign) of every explanatory variable held to be econometrically significant.

As is widely-known, the fixed effects estimator concentrates on the data variation within every statistical unit (in this context, every firm under consideration) and is based entirely on time variation in data[28]. In our analysis, we used this type of estimator for a number of reasons. In the first place, the data available are a closed and complete set of information and the literature shows that in this case fixed effects are the natural candidates as they have the advantage of being able to capture effectively (or to test) all the relevant variables which are idiosyncratic with respect to statistical units which are fixed in time[29]. In the second place, our data show a good variation in the time dimension (14 years) to be able to justify the use of a within estimator. Thirdly, the Least Squares with Dummy Variables (LSDV) estimation method, which is the procedure in the fixed effects context, is BLUE (i.e. the Best Linear Unbiased Estimator) if (i) the model is really of the type $y_{it} = a + b\, x_{it} + \Sigma_{j=1, N-1}\, \mu_j\, D_{ji} + \varepsilon_{it}$, (ii) x is slightly exogenous and if ε_{it} ~IID $(0, \sigma^2_\varepsilon)$ and (iii) it is in any case consistent even though the true model is a random effects model.

Let $x^{(J)}_{it}$ be the *j-simo* regressor in the model while y_{it} represents the default probability associated to each statistical unit, such as $y_{it} = 1$ if the firm is defaulted and $y_{it} = 0$ if the firm is non-defaulted; then in our context of analysis the equation to estimate is the following:

$$y_{it} = a + \sum\nolimits_{j=1,N} b_j x_{it}^{(j)} + \varepsilon_{it} \tag{1}$$

where the error term is equal to a fixed effect plus a true idiosyncratic term $\varepsilon_{it} = \mu_i + w_{it.}$

The fixed effect μ_i “absorbs” all the variables which are fixed in time and every other fixed time factor which may be relevant and which has not been considered explicitly.

The following parameter values are obtained[30]:

[27] Stock *et al.* (2008).

[28] On the opposing side, some estimators are available to analyze the spatial variation of data using separate regression on cross-sectional data (between estimator). The latter can be interpreted as a weighted average of cross-sectional estimation separately conducted. There is also a casual effect estimator which can capture data variations in both dimensions and it is a weighted average of within and between estimators (Baltagi, 2005).

[29] Baltagi, 2005.

[30] As we can note in Table 4, the fixed effects estimator gives the best results with respect to the random effect (as also confirmed by the Hausman test) or the OLS pooled estimator.

Table 4. Results obtained from a fixed-effects estimator on panel data
Dependent variable: firm status

Regressors	**Parameters (Standard Error)**
Lv	-0.101079* (0.022)
CPN	-0.126173* (0.015)
ROI	-0.473325* (0.064)
CONSTANT	0.2290819* (0.013)
* = each parameter is significant to 5% with reference to a bilateral test Number of observations = 2184 Number of groups = 156 Observations for each group = 14 $R^2 = 0.50$ Corr(u_i xb) = 0.579 F(3, 2025) = 110.72; Prob>F = 0.000	sigma_u = 0.1292448 sigma_e = 0.17589481 rho = 0.350061074 (fraction of the variance due to u_i) Test F:all the $u_i = 0$; F(155, 2025) = 7.52 ; Prob > F = 0.000

As can be seen, the variables are the same as those chosen in Section. 4.3.

Our model therefore becomes

$$y = 0.2290819 - 0.1010794\, x^{(1)} - 0.1261728\, x^{(2)} - 0.4733249\, x^{(3)}$$

where $x^{(1)} = \text{Lv}$, $x^{(2)} = \text{CPN}$ and $x^{(3)} = \text{ROI}$.

In order to define the threshold level y_0 which is necessary to distinguish non-defaulting from defaulting companies, we calculated for each of the fourteen years the averages of the values of Lv, CNP and ROI in relation to the firms which had a trading loss even if only for a single year. The choice was made to insert into the y function the lower value obtain within each of the three time series calculated in this way, in an averagely cautious perspective; in this way the values 0.46, 0.34 and -0.002 were pinpointed for the Lv, CPN and ROI variables respectively and we therefore arrived at a value of approximately 0.86 for the cut off.

The prediction error obtained with this procedure is 19.9% overall (32.2% type I error and 16.8% type II error), showing a type I error, the most worrying, which is noticeably more modest than that which is obtained using the same information from Altman's original model, from Altman's "80-20 sample" model and from the use of variables we selected in a discriminant analysis on this latter sample.

An additional advantage of this procedure is the possibility of further reducing type I error, simply by reducing the threshold level, naturally keeping in mind the trade-off between reducing type I error and increasing that of type II which can, in any case, be easily managed.

Therefore, the banking management will define, starting from an objective threshold y_0, by how much to reduce this value keeping in mind the level of risk aversion, the specific nature of the economic context in which the bank is operating and the intuition of its own managers as well as any binding regulations which might exist.

Table 5. Sensitivity analysis: Type I and Type II errors varying from the cut off

cut off: 0.86				*cut off: 0.90*			
	Predicted Group				*Predicted Group*		
Actual Group	non-defaulting	defaulting	Total	*Actual Group*	non-defaulting	defaulting	Total
non- defaulting	104	21	125	non- defaulting	88	37	125
defaulting	10	21	31	defaulting	8	23	31
non- defaulting	83.2	16.8	100.0	non- defaulting	70.4	29.6	100.0
defaulting	32.2	67.8	100.0	defaulting	25.8	74.2	100.0
Overall classification error: 19.9%				*Overall classification error: 28.8%*			

5. Conclusions

The use of the fixed effects panel regression on medium-large sized enterprises set from the Marche region allows a good assessment of bankruptcy risk in terms of type I and type II errors, with a suitable coefficient statistical significativity and a better assessment than that which can be obtained using Altman-style discriminant analysis or its variants.

These results were obtained in stages. The first stage was a re-proposal of Altman's discriminant analysis on our data set with modifications to the numerosity of the subsamples of non-defaulting and defaulting enterprises in relation to the population of firms analyzed. The second step saw the use of new variables and the relative coefficient estimations of the discriminant functions in the new space-time context. Finally, a fixed effect panel linear regression was carried out alongside the linear discriminant analysis from which the factors able to influence critical aspects of business management were progressively enucleated, using independent variables which are different from those in Altman's models.

In selecting these variables, their initial normalization, the level of multicollinearity shown, the degree of statistical significativity and coherence in terms of economic causality were all considered.

It emerges from these results that

- the explanatory variables in the probability of bankruptcy in our context of reference are the normalized values of leverage (Lv), defined as the ratio of net worth plus funded debt to net worth, of the index of asset solidity (CPN), expressed as the ratio of registered capital to net worth and of the index of return on investments (ROI), i.e. the ratio of operating profit to total assets;
- the prediction error is 19.9% overall (32% type I error and 16.8% type II error). Type I error, the most worrying, is noticeably more modest that obtained by using the same information in Altman's original model.

In the method we have put forward, there is a level of arbitrariness relating to the preselected cut off level which arises from the average value of the explanatory variables deduced from enterprises whose balance sheets show a loss. This clearly shows an element of flexibility which permits us to insert operativeness margins into the bankruptcy risk assessment by means of a sensitivity analysis.

An unfavorable situation is undoubtedly adequately addressed by the explicative variables suggested. The drop in ROI and, therefore consequently, the drop in net worth, dwindling reserves, increase the risk of the firm's defaulting. Long-term debts, just like those of the short term variety, potentially offered by the banking systems or other sources of finance may be reduced so creating an increased estimated risk of the firm's insolvency. On this issue, there is a further element which must be introduced into the analysis. This comes out of the reviewing of credit channels and the way they operate in financial crises[31]. In this context constraints on the various intermediaries in terms of their asset solidity, as it is possible to note in their balance sheets, emerge which are relevant for defining credit supply conditions[32]. In this perspective, a functional link can be established between the cut off level and the level of resilience/fragility of the banking system which may, in situations of difficulty, increase rigor in the assessment of credit merit. In terms of this model, this means implementing a management and organizational strategy which leads to the reduction of type I error while increasing type II error. The banking system's change of strategy increases uncertainty for a greater number of enterprises with the potential consequence that it reduces the finance availability from outside the production system. Endogenizing the cut off level is an issue to be placed on the agenda of the current research project parallel to lengthening the time factor in this panel data. This will allow us to move on to further testing of the model and to achieve a better out of the sample calibration.

[31] Bank for International Settlements, 2010.
[32] Del Giovane *et al.*, 2010.

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